



# JEE Advanced Home Practice Test -1 | Paper -1 | JEE 2024

Date: 20/04/2024 Maximum Marks: 198

**Duration: 3.0 Hours** 

## **General Instructions**

- 1. The question paper consists of 3 Subject (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Part has **THREE** sections (Section 1, Section 2 & Section 3).
- **Section 1** contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four option(s) is (are) correct answer(s).
- 3. Section 2 contains SIX (06) questions. Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- **4. Section 3** contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. *In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, <i>Θ sign should be bubbled.* (Example: 6, 81, 1.50, 3.25, 0.08)
- **5.** For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code**, **Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

| Name of the Candidate (In CAPITALS) :          |
|--|
| Roll Number:                                   |
| OMR Bar Code Number :                          |
| Candidate's Signature: Invigilator's Signature |

### MARKING SCHEME

#### **SECTION-1**

- This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);

Negative Marks: -1 In all other cases.

#### **SECTION-2**

• This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both

of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen, both of

which are correct;

Zero Marks : 0 If unanswered; Negative Marks : -2 In all other cases.

### **SECTION-3**

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct **NUMERICAL VALUE** of the answer. If the answer is a decimal numerical value, then round-off the value to **TWO** decimal places. If the answer is an Integer value, then do not add zero in the decimal places. In the OMR, do not bubble the ⊕ sign for positive values. However, for negative values, Θ sign should be bubbled. (Example: 6, 81, 1.50, 3.25, 0.08)
- Answer to each question will be evaluated according to the following marking scheme:

**Full Marks** : +4 If ONLY the correct numerical value is entered;

**Zero Marks**: 0 In all other cases.

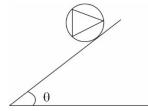
### **SUBJECT I: PHYSICS**

66 MARKS

#### **SECTION-1**

This contains SIX (06) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four option(s) is (are) correct answer(s).

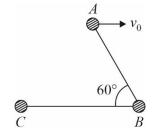
1. The figure shows a light ring with three rods, each of mass m welded on this ring. The rods form an equilateral triangle. The rigid assembly is released on a rough fixed inclined plane. Determine the minimum value of the coefficient of static friction, that will allow pure rolling of the assembly.



- (A)  $\frac{2}{3}\tan\theta$  (B)  $\frac{1}{2}\tan\theta$
- **(C)**  $\tan \theta$
- (D)  $\frac{4}{2}\tan\theta$
- 2. A steel drill is making 180 revolutions per minute, under a constant torque of 5N-m. If it drills a hole in 7 sec in a steel block of mass 600 gm, rise in temperature of the block is:

(For steel  $S = 0.1 cal \ gm^{-1} \ ^{0}C^{-1}$ ) (1 cal = 4.2 J) (Neglect heat lost to drill and surrounding)

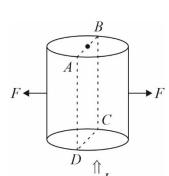
- (A) 2.6°C
- 1.3°C **(B)**
- **(C)** 5.2°C
- $3^{\circ}C$ **(D)**
- 3. Three blocks A, B and C of masses 1kg, 1kg and 2kg respectively are kept on a smooth horizontal surface. Top view of the same is drawn below. Initially strings connecting the three blocks are just taught (Tension = 0). Initially block A is given a velocity  $v_0$  parallel to the string connecting block B and C. What is the initial non-zero velocity of block C during the motion?



**(A)** 

(C)  $\frac{7}{2}v_0$ 

- 4. An infinitely long solid cylinder of radius 'a' carries current uniformly across the cross-section with current density J along its length. If the cylinder is cut in two equal parts along its length by plane ABCD as shown. Force per unit length required to separate the two halves will be:



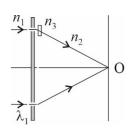
(A)

 $\frac{2\mu_0 J^2 a^3}{5}$ 

 $\frac{2\mu_0 J^2 a^2}{2}$ **(C)** 

**(D)**  $\mu_0 J^2 a^3$ 

5. In YDSE (as shown in the figure), a parallel beam of light is incident on the slit from a medium of refractive index  $n_1$ . The wavelength of light in this medium is  $\lambda_1$ . A transparent slab of thickness 't' and refractive index  $n_3$  is put in front of one slit. The medium between the screen and the plane of the slits in  $n_2$ . The phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits) is:

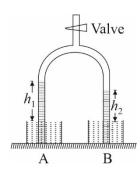


 $(\mathbf{A}) \qquad \frac{2\pi}{n_1\lambda_1} (n_3 - n_2)t$ 

 $\mathbf{(B)} \qquad \frac{2\pi}{\lambda_1} (n_3 - n_2) t$ 

 $(\mathbf{C}) \qquad \frac{2\pi n_1}{n_2 \lambda_1} \left( \frac{n_3}{n_2} - 1 \right) t$ 

- **(D)**  $\frac{2\pi n_1}{\lambda_1} (n_3 n_1)t$
- 6. The limbs of a glass U-tube are lowered into vessels A and B as shown in figure. Some air is pumped out through a valve, placed at the top of tube and then the valve is closed. The liquid in the left-hand limb then rises to  $h_1 = 20 \, cm$  and in the right hand it rises to a height  $h_2 = 10 \, cm$ . Vessel A has water, then the density of liquid in vessel B is:



(A)  $2 \times 10^3 kg / m^3$ 

**(B)**  $3 \times 10^3 kg / m^3$ 

(C)  $4 \times 10^3 kg / m^3$ 

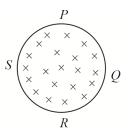
**(D)**  $4.5 \times 10^3 kg / m^3$ 

#### **SECTION-2**

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

7. Suppose the potential energy between electron and proton at a distance r is given by  $-\frac{Ke^2}{2r^3}$ . Using Bohr's theory choose the correct statements:

- (A) Energy in the nth orbit is proportional to  $n^3$
- **(B)** Energy in the nth orbit is proportional to  $n^6$
- (C) Energy is proportional to  $m^2(m : mass of electron)$
- **(D)** Energy is proportional to  $m^{-3}$  (m: mass of electron)
- 8. A conducting ring is placed in a uniform external magnetic field present in the space within the ring and perpendicular to plane of ring. The magnetic field is increasing at constant rate due to which a current of magnitude 4A amp flows in the ring. The resistance of parts PQR and PSR are  $4\Omega$  and  $8\Omega$  respectively. Then choose the correct statements: (P & R are diametrically opposite points)



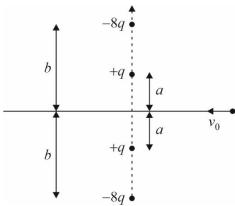
- (A) The emf induced in the ring 48V
- **(B)** Potential difference between P & R is 24V
- (C) Potential difference between P & R is 8V
- **(D)** Potential difference between P & R is 40V

**9.** The velocity, acceleration and force in two systems of units are related as under:

(i) 
$$v' = \frac{\alpha^2}{\beta}v$$
 (ii)  $a' = (\alpha \beta)a$  (iii)  $F' = \left[\frac{1}{\alpha\beta}\right]F$ 

All the primed symbols belong to one system and unprimed ones belong to the other system.  $\alpha$  and  $\beta$  are dimensionless constants. Which of the following are correct?

- (A) Length standards of the two system are related by:  $L' = \frac{\alpha^3}{\beta^3} L$
- **(B)** Mass standards of the two systems are related by:  $M' = \left(\frac{1}{\alpha^2 \beta^2}\right) M$
- (C) Time standards of the two systems are related by:  $T' = \left(\frac{\alpha}{\beta^2}\right)T$
- **(D)** Momentum standards of the two systems are related by:  $p' = \left(\frac{1}{\beta^3}\right)p$
- 10. Charges -8q, -8q, q, q are fixed on the y-axis as shown. A point charge (q, m) is projected towards origin along x-axis from a very large distance with speed  $v_0$  in order to reach -ve infinity. Choose the correct option(s).



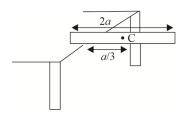
- (A) If b = 2.5a there will be two points where force on q will be zero
- **(B)** Minimum potential energy of charge q is  $\frac{-3\sqrt{3} q^2}{2\pi \in_0 \sqrt{b^2 a^2}}$
- (C) If b = 4a, minimum value of  $v_0$  is  $\sqrt{\frac{q^2}{\pi \in_0 ma}}$
- **(D)** If b = 16a, minimum value of  $v_0$  is  $\sqrt{\frac{q^2}{4\pi \in_0 ma}}$

- 11. A wire 5.8m long, 2mm in diameter carries 750mA current when 22mV potential difference is applied at its ends. If drift speed of electrons is found  $1.7 \times 10^{-5} m/s$ , then choose the correct options.
  - Current density is  $2.4 \times 10^5 A/m^2$ **(A)**
  - Current density is  $1.6 \times 10^{-4} A / m^2$ **(B)**
  - Number of free electrons per unit volume is  $2.7 \times 10^{16} \, m^{-3}$ **(C)**
  - Number of free electrons per unit volume is  $8.8 \times 10^{28} \, m^{-3}$ **(D)**
- Two identical containers each of volume  $V_0$  are joined by a small pipe. The containers contain identical 12. gases at temperature  $T_0$  and pressure  $P_0$ . One container is heated to temperature  $2T_0$  while maintaining the other at the same temperature. The common pressure of the gas is P and n is the number of moles of gas in the container at temperature  $2T_0$ . Then:
- (A)  $P = 2P_0$  (B)  $P = \frac{4}{3}P_0$  (C)  $n = \frac{2}{3}\frac{P_0V_0}{RT_0}$  (D)  $n = \frac{3}{2}\frac{P_0V_0}{RT_0}$

### **SECTION-3**

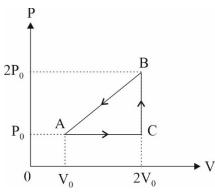
This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. In the OMR, do not bubble the  $\oplus$  sign for positive values. However, for negative values,  $\Theta$  sign should be bubbled. (Example: 6, 81, 1.50, 3.25, 0.08).

A uniform rod of length 2a is placed horizontally on the edge of a table. 13. Initially, the centre of mass of the rod is at a distance  $\frac{a}{2}$  from the edge. The rod is released from rest. If the rod slips after it has turned through an angle  $\theta$ , the coefficient of friction between the rod and the table is  $n \tan \theta$ . Find the value of n.

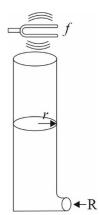


- 14. A mercury drop shaped as a round tablet of radius R and thickness h is located between two horizontal glass plates. Assume that  $h \ll R$ , the contact angle equals  $\theta$  (which is obtuse) and surface tension is T. The mass m of a weight which has to be placed on the upper plate to diminish the distance between the plates *n*-times is  $m = \frac{k\pi R^2 T \cos \theta (1 - n^2)}{\sigma h}$ . Find the value of *k*.
- 15. A non-conducting ring of mass m and radius r is lying at rest in the vertical xy plane on a smooth non-conducting horizontal xz plane. Charges +q and -q are distributed uniformaly on the ring on the two sides of the vertical diameter of the ring. A constant and uniform electric field E is set up along the x-direction. The ring is given a small rotation about the vertical diameter of the ring and released. The period of oscillation of the ring is  $\frac{\pi}{k} \sqrt{\frac{m\pi r}{2aE}}$ . Find the value of k.

16. One mole of a monatomic gas is taken from a point A to another point B along the path ACB. The initial temperature at A is  $T_0$ . The heat absorbed by the gas in the process  $A \to C \to B$  is  $xRT_0$ . Find the value of x.



In the given figure, water is being pumped into a tall, vertical cylinder at a volume flow rate R. The radius of the cylinder is r, and at the open top of the cylinder a tuning fork is vibrating with a frequency f and v is the velocity of sound wave. As the water rises, the time interval elapses between successive resonance is  $\frac{\beta \pi r^2 v}{Rf}$ . Find the value of  $\beta$ .



18. A circular disc of radius R carries surface charge density  $\sigma_0(r) = \sigma_0 \left(1 - \frac{r}{R}\right)$ , where  $\sigma_0$  is a constant and r is the distance from the centre of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is  $\phi_0$ . Electric flux through another spherical surface of radius  $\frac{R}{2}$  and concentric with the disc is  $\phi$ . Then ratio  $\frac{\phi_0}{\phi}$  is \_\_\_\_\_.

### **SUBJECT II: CHEMISTRY**

66 MARKS

#### **SECTION-1**

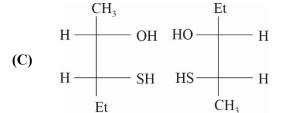
This contains SIX (06) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four option(s) is (are) correct answer(s).

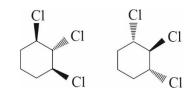
- 1. Select incorrect statement regarding maxwell-Boltzmann distribution of molecular speeds:
  - In case of unsymmetrical distribution curve  $V_{mps} < V_{avg} < V_{rms}$ **(A)**
  - In case of symmetrical distribution curve  $\,V_{mps} < V_{avg} < V_{rms}\,$ **(B)**
  - **(C)** The ratio of the most probable, the average and root mean square speed respectively is 1:1.128:1.224
  - **(D)** Height of the distribution curve decreases while increasing temperature
- 2. Which of the following do not liberate  $O_2$  upon heating?
  - **(A)**  $Pb(NO_3)_2$  (B)  $LiNO_3$  (C)
- BaO<sub>2</sub>
- (D)  $Na_2CO_3$
- When triatomic gas X<sub>3</sub> reacts with an excess of potassium iodide solution buffered with a borate buffer 3. (pH = 9.2), diatomic product  $Y_2$  is liberated which can be titrated against a standard solution of sodium thiosulphate. This is a quantitative method for the estimation of X3 gas in the mixture of  $X_3$  and  $X_2$  of same atom X. When liberated  $Y_2$  reacts with  $Na_2S_2O_3$  it produces (Z). What is Z?
  - **(A)** Na<sub>2</sub>SO<sub>4</sub>
- **(B)**

**(D)** 

- $Na_2SO_4 + S$  (C)  $Na_2S_2O_3$  (D)  $Na_2S_4O_6$
- Which of the following pair is diastereomers? 4.

(B) 
$$C = C \xrightarrow{CH_3} H \xrightarrow{CH_3} C = C \xrightarrow{H} C = C$$





**5.** IUPAC name of the following compound is/are:

- (A) (2-chlorocyclopropenyl)-4-chloro-2-fluoro-3 [2-oxoformyl] cyclohex-5-ene-1-carboxylate
- (B) (2-chlorocyclopropenyl)-4-chloro-6-fluoro-5 [2-oxoethyl] cyclohex-2-ene-1-carboxylate
- (C) (2-chlorocyclopropenyl)-4-chloro-6-fluoro-5 [formylmethyl] cyclohexane-1-carboxylate
- (D) (2-chlorocyclopropenyl)-4-chloro-2-fluoro-3[formylmethyl] cyclohex-5-ene-1-carboxylate
- **6.** Which statement is **incorrect** about following?

- (A) It is sucrose
- **(B)** Due to presence of hemiacetal linkage, it is non-reducing sugar
- (C) Left side unit is  $\alpha$ -D-glucose and right-side unit is  $\beta$ -D-fructose
- (D) Due to absence of hemiacetal linkage, it can't show mutarotation

#### **SECTION-2**

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

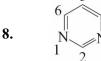
7. In an adiabatic process, the work involved during expansion or compression of an ideal gas is given by:

(A) 
$$nC_v\Delta T$$

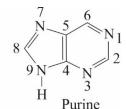
$$(\mathbf{B}) \qquad \frac{nR}{\gamma - 1} \big( T_2 - T_1 \big)$$

(C) 
$$-nRP_{ext}\left[\frac{T_2P_1-T_1P_2}{P_1P_2}\right]$$

$$\gamma - 1 = V_2$$
(D)  $-2.303$ RT  $\log \frac{V_2}{V_1}$ 



 $N_3$   $N_2$   $N_1$ 



Pyrimidine Imidazole

Among the following which statement(s) is/are true?

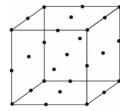
- (A) Both N of pyrimidine are of same basic strength
- **(B)** In imidazole protonation takes place on N-3
- (C) Purine has least basic 'N' at position 9
- **(D)** Pyrimidine, imidazole and purine all are aromatic

**9.** In the reaction scheme shown below Q, R, S, T are the major products.

$$\begin{array}{c|c} O \\ C-H \\ C-H \\ O \end{array} \xrightarrow{\text{dil. NaOH}} Q \xrightarrow{H^+} R \xrightarrow{(i)} O \xrightarrow{O} O \\ (ii) \quad AlCl_3 \longrightarrow S \xrightarrow{(i)} Zn-Hg/HCl} T$$

The correct structure of:

- 10. Which of the following statement(s) is/are true about the complex  $\left[ \text{CrCl}_3 \left( \text{OH} \right)_2 \left( \text{NH}_3 \right) \right]^{2-}$  ion?
  - (A) It has three geometrical isomers
  - **(B)** Only one space isomers is optically active and remaining are inactive
  - (C) There are total four space isomers
  - **(D)** It is paramagnetic complex
- 11. The correct statement(s) about the oxoacids HClO, HClO<sub>3</sub> and HClO<sub>4</sub> is/are:
  - (A)  $HClO_4$  is stronger oxidising agent than  $HClO_3$
  - **(B)** The conjugate base of HClO<sub>4</sub> is weaker base than H<sub>2</sub>O
  - (C) Hypochlorite ion and perchlorate ion undergo disproportionation and give rise to same set of ions
  - **(D)** Hypochlorite ion oxidises nitrite
- 12. In an AB unit cell (Rock salt type) assuming Na<sup>+</sup> forming fcc:



- (A) The nearest neighbour of  $A^+$  is  $6B^-$  ion
- **(B)** The nearest neighbour of  $B^-$  is  $6A^+$  ion
- (C) The second neighbour of  $A^+$  is  $12A^+$
- **(D)** The packing fraction of AB crystal is  $\frac{\sqrt{3\pi}}{8}$

### **SECTION-3**

This section contains **SIX** (**06**) questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. In the *OMR*, do not bubble the  $\oplus$  sign for positive values. However, for negative values,  $\Theta$  sign should be bubbled. (Example: 6, 81, 1.50, 3.25, 0.08).

- A solution containing Na<sub>2</sub>CO<sub>3</sub> and NaOH requires 300 mL of 0.1 N HCl using phenolphthalein as an indicator. Methyl orange is then added to the above titrated solution when a further 25 mL of 0.2 N HCl is required. The amount of NaOH (in gm) present in solution is \_\_\_\_\_.
   (Molecular mass of NaOH = 40, Na<sub>2</sub>CO<sub>3</sub> = 106)
- **14.** For the equilibrium,

$$NiO(s) + CO(g) \rightleftharpoons Ni(s) + CO_2(g), \Delta G^{\circ}(cal/mol) = -5320 - 5.6T(K).$$

The temperature (in Kelvin) at which the gaseous mixture at equilibrium contains 400ppm of CO by mole is  $[\ln 10 = 2.3, \ln 2 = 0.7] \text{ (Assume } 10^6 - 400 = 10^6)$ 

15. Consider 80% efficient hydrogen-oxygen fuel cell working under standard conditions at 1 bar and 298K. Its cell reaction is  $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell)$ 

The work derived from the cell on the consumption of 2.27L of  $H_2(g)$  is used to compress 1.00 mol of a  $CO_2$  gas in a thermally insulated container. What is the change in the temperature (in K) of the ideal gas? The standard reduction potentials for the two half-cells are given below.

$$O_2(g) + 4H^+(aq) + 4e^- \longrightarrow 2H_2O(\ell), E^\circ = 1.23V,$$

$$2H^{+}(aq) + 2e^{-} \longrightarrow H_{2}(g), E^{0} = 0.00V.$$

Use 
$$F = 96500 \text{ C mol}^{-1}$$
,  $R = 8.314 \text{ J mol}^{-1} \text{K}^{-1}$ .

16. Magnesium reacts with sulfuric acid to form magnesium sulphate and hydrogen. What is the volume of hydrogen gas in litre produced at STP when 4.8 gm of magnesium and 50.0 mL of 5.0M sulfuric acid are combined for the reaction?

(Use molar mass of magnesium as  $24.0g\ mol^{-1}$ ,  $R = 0.082atm\ L\ mol^{-1}K^{-1}$ )

17.  $_{84}\text{Po}^{218}\left(t_{1/2}=183\,\text{sec}\right)$  decay to  $_{82}\text{Pb}\left(t_{1/2}=161\,\text{sec}\right)$  by  $\alpha$ -emission, while Pb<sup>214</sup> is a  $\beta$ -emitter. In an experiment starting with 1 mole of pure Po<sup>218</sup>, how much time would be required for the number of nuclei of  $_{82}\text{Pb}^{214}$  to reach maximum?

**18.** Consider following sequence of reactions.

$$O \longrightarrow O \xrightarrow{H_3O^+} A + B$$

$$C \xrightarrow{\text{sodalime}} D$$

$$B \xrightarrow{OH/\Delta} E$$

Total sum of degree of unsaturations of product D and E will be \_\_\_\_\_.

### **SUBJECT III: MATHEMATICS**

66 MARKS

#### **SECTION-1**

This contains SIX (06) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four option(s) is (are) correct answer(s).

- Suppose a, b denotes the distinct real roots of the quadratic equation  $x^2 + 22x 2020 = 0$  and suppose 1. c, d denote the distinct complex roots of the quadratic equation  $x^2 - 22x + 2020 = 0$ . Then the value of ac(c+a)+ad(a+d)+bc(c+b)+bd(b+d) is:
  - **(A)** 18860
- **(B)** 808060
- **(C)** 177760
- **(D)** 160000
- If Rolle's theorem is applicable to the function  $f(x) = x^3 3x^2 x + 1$  in [a, b],  $a \ne b$ , where a and b 2. are integers and  $-1 \le a \le 3$  and  $-1 \le b \le 3$ , then the sum of all possible values of a + b is:
  - 6 (A)
- 8 **(B)**
- **(C)** 12
- **(D)** 16
- 3. Consider a paper in the shape of an equilateral triangle ABC with circumcentre O and perimeter 9 units. If we fold the paper in such a way that each of the vertices A, B, C gets coincided with O, then the area of the resulting hexagonal shape in square units is:
  - (A)
- (B)  $\frac{4}{\sqrt{2}}$  (C)  $\frac{3\sqrt{3}}{2}$
- **(D)**  $3\sqrt{3}$
- 4. Chords AB and CD of a circle intersect at right angle at the point P. If the lengths of AP, PB, CP are 2, 6, 3 units respectively, then the radius of the circle is:
  - (A)

- (B)  $\frac{\sqrt{65}}{2}$  (C)  $\frac{\sqrt{66}}{2}$  (D)  $\frac{\sqrt{67}}{2}$
- Suppose that the foci of the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  are  $(f_1, 0)$  and  $(f_2, 0)$  where  $f_1 > 0$  and  $f_2 < 0$ . Let 5.  $P_1$  and  $P_2$  be two parabolas with a common vertex at (0,0) with foci at  $(f_1,0)$  and  $(2f_2,0)$ , respectively. Let  $T_1$  be a tangent to  $P_1$  which passes through  $(2f_2, 0)$  and  $T_2$  be a tangent to  $P_2$  which passes through  $(f_1, 0)$ . Let  $m_1$  is the slope of  $T_1$  and  $m_2$  is the slope of  $T_2$ , then the value of
  - $\left(\frac{1}{m_1^2} + m_2^2\right)$  is:
  - (A)
- **(B)** 3
- **(C)**
- **(D)** 9

- Let  $0 < x < \frac{1}{6}$  be a real number. When a certain biased dice is rolled, a particular face F occurs with 6. probability  $\frac{1}{6} - x$  and its opposite face occurs with probability  $\frac{1}{6} + x$  and all other be equally likely. Recall that the opposite faces sum is 7 in any dice. Assume that the probability of obtaining the sum 7 when two such dice are rolled is  $\frac{13}{96}$ . Then the value of x is:
  - **(A)**

- (B)  $\frac{1}{12}$  (C)  $\frac{1}{24}$  (D)  $\frac{1}{27}$

#### **SECTION-2**

This section contains SIX (06) questions. Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN **ONE** of these four option(s) is (are) correct answer(s).

- If f(x) and g(x) are differentiable functions for  $0 \le x \le 1$  such that f(0) = 2, g(0) = 0, 7. f(1) = 6, g(1) = 2, then which of the following are true for some 0 < c < 1 (c in one option may be different from c in another)?
  - f'(c)-f(0)=g'(c)(A)
- **(B)** f'(c) g(0) = 2g'(c)
- f'(c) + f(1) = 3g'(c)**(C)**
- **(D)** f'(c) + 2g(1) = 4g'(c)
- 8. Let M be a  $3 \times 3$  invertible matrix and I be a  $3 \times 3$  identity matrix, then which of the following is/are true?
  - (A)
    - $adj(adj(adj(M))) = |M|^2 adj(M)$  (B) adj(adj(adj(M))) = |M|adj(M)
  - $|M|M^{-1}adj(M^{-1}) = I$ **(C)**
- **(D)**  $adj((kM)^{-1}) = \frac{1}{L^2}(adj(M^{-1})), k \in R \{0\}$
- Consider the following two subsets of C,  $A = \left\{ \frac{1}{z} : |z| = 2 \right\}$  and  $B = \left\{ \frac{1}{z} : |z 1| = 2 \right\}$ . Which of the 9. following is/are correct?
  - A is a circle with radius  $\frac{1}{2}$ **(A)**
- **(B)** B is a circle with centre  $\left(-\frac{1}{3}, 0\right)$
- B is a circle with radius  $\frac{2}{2}$ **(C)**
- **(D)** A is circle but B is not a circle
- In a triangle,  $a^2 + b^2 + c^2 = ca + ab\sqrt{3}$ , then in the triangle ABC which of the following is **NOT** 10. **CORRECT**? (a, b, c are sides opposite to angles A, B, C respectively)

- (A)  $A = 30^{\circ}, B = 60^{\circ}, C = 90^{\circ}$
- **(B)**  $A = 60^{\circ}, B = 30^{\circ}, C = 90^{\circ}$
- **(C)**  $A = 90^{\circ}, B = 60^{\circ}, C = 30^{\circ}$
- **(D)**  $A = 90^{\circ}, B = 45^{\circ}, C = 45^{\circ}$

- 11. If three planes  $P_1 = 2x + y + z 1 = 0$ ,  $P_2 = x y + z 2 = 0$  and  $P_3 = \alpha x y + 3z 5 = 0$  intersects each other at point P on XOY plane and at point Q on YOZ plane, where Q is the origin then identify the correct statement(s)?
  - (A) The value of  $\alpha$  is 4
  - **(B)** Straight line perpendicular to plane  $P_3$  and passing through P is  $\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z}{3}$
  - (C) The length of projection of  $\overrightarrow{PQ}$  on x-axis is 1
  - **(D)** Centroid of the triangle OPQ is  $\left(\frac{1}{3}, \frac{-1}{2}, \frac{1}{2}\right)$
- 12. For  $a \ge 1$ , let  $I(a) = \int_{-1/a}^{1/a} (2x^6 + 2x^4 + 3)\cos^{-1}(ax) dx$ , then which of the following is/are correct?
  - $(\mathbf{A}) \qquad I(a) \le \frac{129\pi}{35a}$
- **(B)**  $I(a) = \pi \left(\frac{2}{7a^7} + \frac{2}{5a^5} + \frac{3}{a}\right)$
- (C) I(a) is independent of a
- **(D)** I(1) = 0

#### **SECTION-3**

This section contains **SIX** (06) questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places. If the answer is an Integer value, then do not add zero in the decimal places. In the *OMR*, do not bubble the  $\oplus$  sign for positive values. However, for negative values,  $\Theta$  sign should be bubbled. (Example: 6, 81, 1.50, 3.25, 0.08).

- 13. Assume that *n* copies of the unit cube are glued together side by side to form a rectangular solid block. If the number of unit cubes that are completely invisible is 30, then the minimum possible value of *n* is \_\_\_\_\_\_.
- 14. If the value of  ${}^{200}C_0 {}^{200}C_1 + {}^{200}C_2 {}^{200}C_3 + \dots$   ${}^{200}C_{148}$  is equal to  ${}^xC_y$  then least value of x+y is \_\_\_\_\_\_.
- 15. For  $0 \le x < 2\pi$ , the number of solutions of the equation  $\sin^2 x + 2\cos^2 x + 3\sin x \cos x = 0$  is \_\_\_\_\_.
- 16. Six consecutive sides of an equiangular octagon are 6, 9, 8, 7, 10, 5 in that order. The integer nearest to the sum of the remaining two sides is \_\_\_\_\_.
- 17. Let  $f: R \to R$  be a function defined by  $f(x) = \begin{cases} x^3 + 2x^2 + x + c, & \text{if } x \le b \\ e^x, & \text{if } x > b \end{cases}$ , where b and c are integers. If f(x) is differentiable  $\forall x \in R$ , then the value of (b+c) is \_\_\_\_\_.
- 18. Let  $f(x) = \frac{1}{2}x\sin x (1-\cos x)$ . For the smallest positive integer k such that  $\lim_{x\to 0} \frac{f(x)}{x^k} = \lambda$ ,  $\lambda \neq 0$  then  $k + 24\lambda =$